Spin-orbit scattering in superconducting nanoparticles

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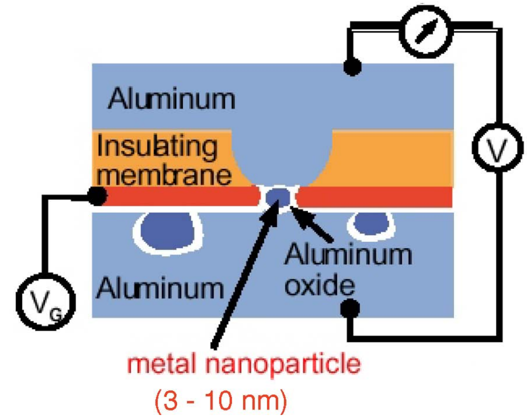
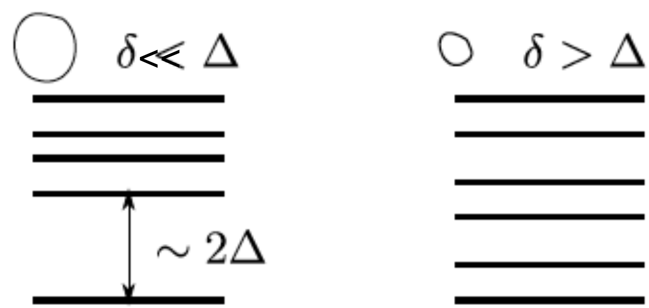


- Discrete energy levels extracted from non-linear conductance measurements (Ralph et al).
- Experiments on Al, Co, Au, Cu and Ag.

Nano-scale grains: probe the quantum regime $T << \delta$

Superconducting grains: materials that are superconductors in the bulk and characterized by a pairing gap Δ.

δ = single-particle level spacing.



BCS regime

Fluctuation-dominated regime
(BCS theory breaks down)

Can we observe signatures of pairing correlations in the fluctuation-dominated regime ?

Hamiltonian

Assume a nanoparticle with chaotic single-electron dynamics:

- Single-particle Hamiltonian is described by random-matrix theory
- Interactions are determined by symmetry considerations

I. Without spin-orbit scattering

- Spin-rotation invariance plus time-reversal symmetry (TRS)

  $\Rightarrow$ Spin-degenerate single-particle levels ε_i described by GOE

$$H = \sum_i \varepsilon_i (a_{i\uparrow}^\dagger a_{i\uparrow} + a_{i\downarrow}^\dagger a_{i\downarrow}) + \frac{e^2}{2C} N^2 - G P^\dagger P - J_s \vec{S}^2 \qquad (P^\dagger = \sum_i a_{i\uparrow}^\dagger a_{i\downarrow}^\dagger)$$

- Pairing plus ferromagnetic exchange interactions ($\vec{S}$ is the total spin)

II. With spin-orbit scattering

- Breaks spin-rotation symmetry but preserves TRS

  $\Rightarrow$ Kramers-degenerate (1,2) single-particle levels ε_i described by GSE

$$H = \sum_i \varepsilon_i (a_{i1}^\dagger a_{i1} + a_{i2}^\dagger a_{i2}) + \frac{e^2}{2C} N^2 - G P^\dagger P \qquad (P^\dagger = \sum_i a_{i1}^\dagger a_{i2}^\dagger)$$

- Exchange interaction is suppressed

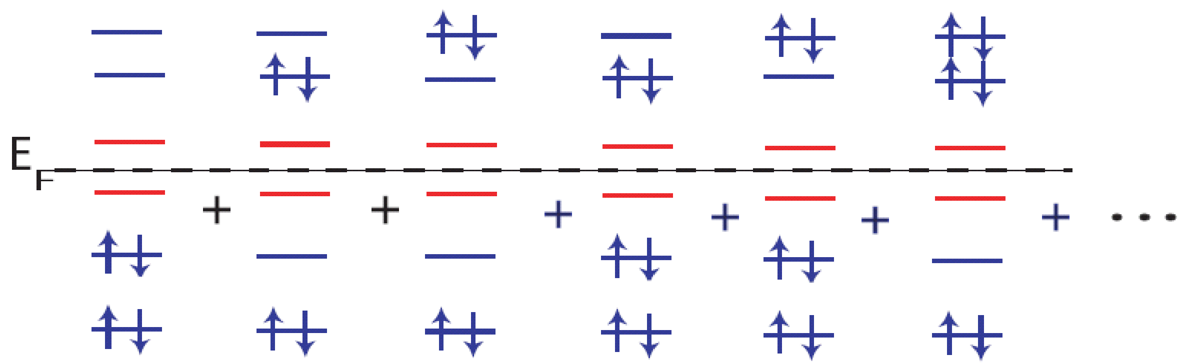
Solution - method 1: eigenstates

The eigenstates are $|U\varsigma; B\gamma SM>$

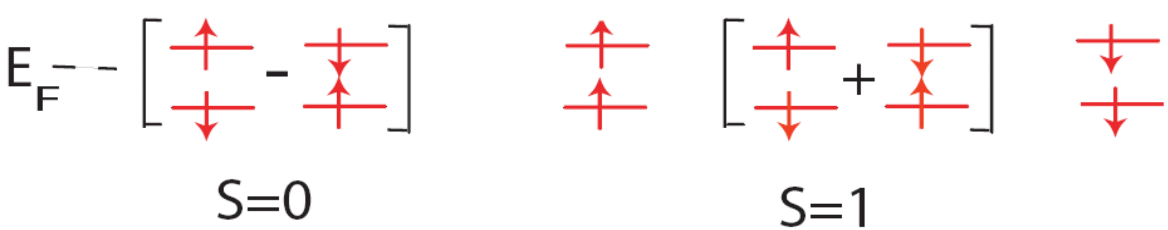
U is a subset of doubly occupied and empty levels.
B is a subset of singly occupied levels

(i) $|U\varsigma>$ are zero-spin eigenstates of the reduced pairing Hamiltonian (Richardson)

Singly occupied levels (red) are *blocked* with respect to pairing



(ii) $|B\gamma SM>$ are eigenstates of S^2, obtained by coupling spin-1/2 singly occupied levels in B to total spin S and spin projection M .



Solution - method 2: partition function

$$H = H_P - J_s \vec{S}^2$$ (H_P is the reduced pairing Hamiltonian)

(i) Exchange: use exact spin projection method

$$Tr e^{-\beta H} = \sum_S e^{\beta J_s S(S+1)} Tr_S e^{-\beta H_P}$$

Trace over states with fixed spin S

(ii) Pairing: Hubbard-Stratonovich transformation for H_P :

$$e^{-\beta H_P} = \int D[\Delta(\tau), \Delta^*(\tau)] T e^{-\int_0^\beta d\tau (|\Delta(\tau)|^2/G + h[\Delta(\tau),\Delta^*(\tau)])}$$

one-body Hamiltonian in pairing field $\Delta(\tau)$

Expand $\Delta(\tau) = \Delta_0 + \sum_m \Delta_m e^{i\omega_m \tau}$ (ω_m are Matsubara frequencies).

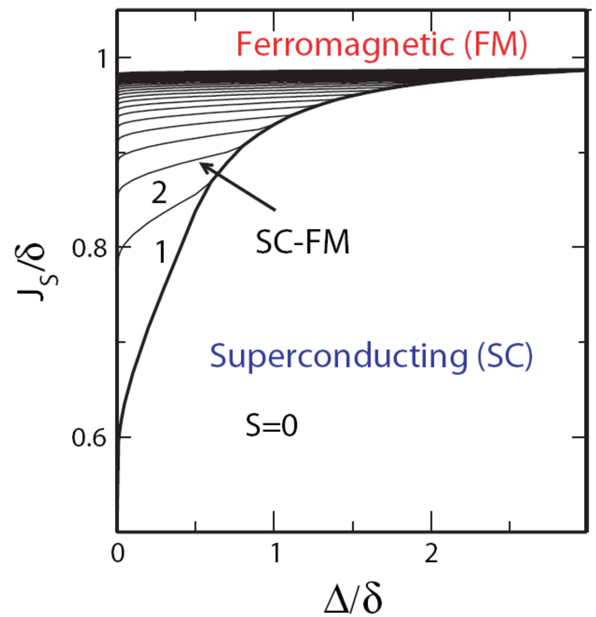
Integrate over static fluctuations Δ_0 exactly and over Δ_m by saddle point (i.e., small-amplitude quantal fluctuations) around each static Δ_0

(iii) Number-parity projection to capture odd-even effects $P_\pm = (1 \pm e^{i\pi N})/2$

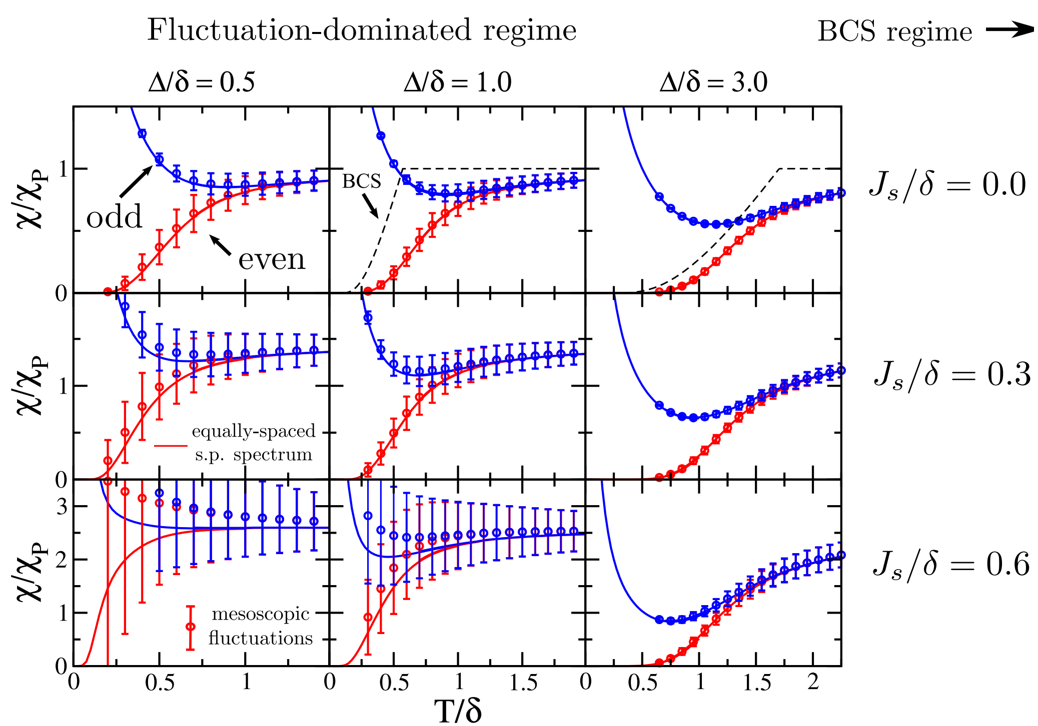
Results: no spin-orbit scattering

Phase diagram of the ground-state spin

A coexistence regime of superconducting and ferromagnetic correlations ($S \neq 0$).



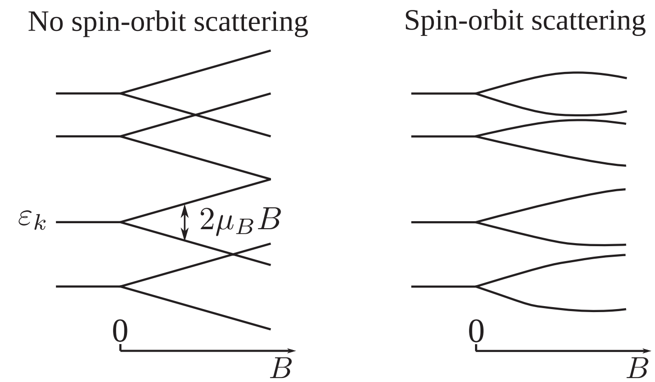
Spin susceptibility



- BCS regime: exchange correlations enhance the odd re-entrant effect.
- Fluctuation-dominated regime: exchange correlations enhance the fluctuations of the susceptibility.

Results: with spin-orbit scattering

Many-body levels of the odd nanoparticle are doubly degenerate (TRS) and they split in a magnetic field



$$\Delta E = \Delta E(0) \pm \frac{1}{2} g \mu_B B + \frac{1}{2} \kappa B^2$$

(i) g factor

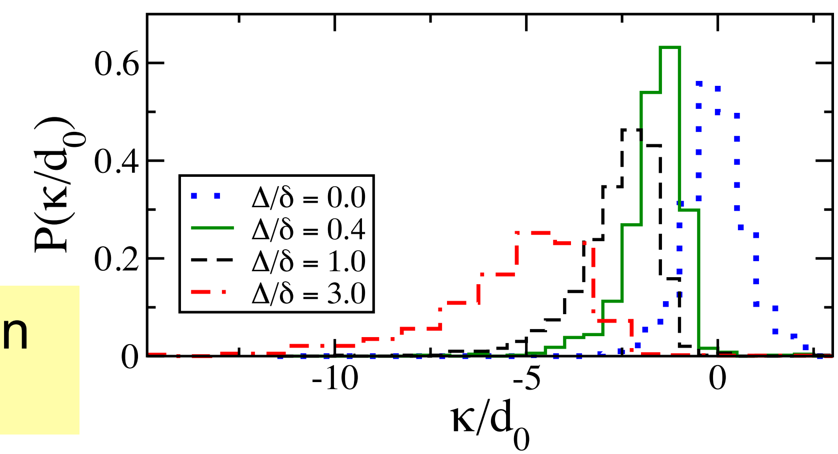
even ground state: $\left\langle C_1 + C_2 + \ldots \middle| \hat{M}_z \middle| C_1 + C_2 + \ldots \right\rangle = 0$ by TRS ($\hat{M}_z$ is odd)

odd ground state: $\left\langle C_1' + C_2' + \ldots \middle| \hat{M}_z \middle| C_1' + C_2' + \ldots \right\rangle = \left\langle \cdot \middle| \hat{M}_z \middle| \cdot \right\rangle_{\text{single-particle}}$ by TRS and blocking

$\Longrightarrow$ g-factor statistic is not affected by pairing

(ii) Level curvature κ

Level curvature distribution is highly sensitive to pairing



$\Longrightarrow$ A tool to probe pairing correlations in the SET spectroscopy experiments.

Conclusions

- A superconducting nano-scale metallic grain is characterize by two regimes: BCS regime $\Delta / \delta >> 1$ and fluctuation-dominated regime $\Delta / \delta \leq 1$

I. In the absence of spin-orbit scattering:

- Coexistence of superconductivity and ferromagnetism in the fluctuation-dominated regime
- Effects of exchange correlations on the odd-even signatures of pairing correlations are qualitatively different in the BCS and fluctuation-dominated regimes

II. In the presence of spin-orbit scattering:

- Spin exchange correlations are suppressed
- g-factor statistic is unaffected by pairing correlations
- Level curvature statistic is highly sensitive to pairing correlations