Low-energy enhancement in the γ-ray strength functions of heavy nuclei

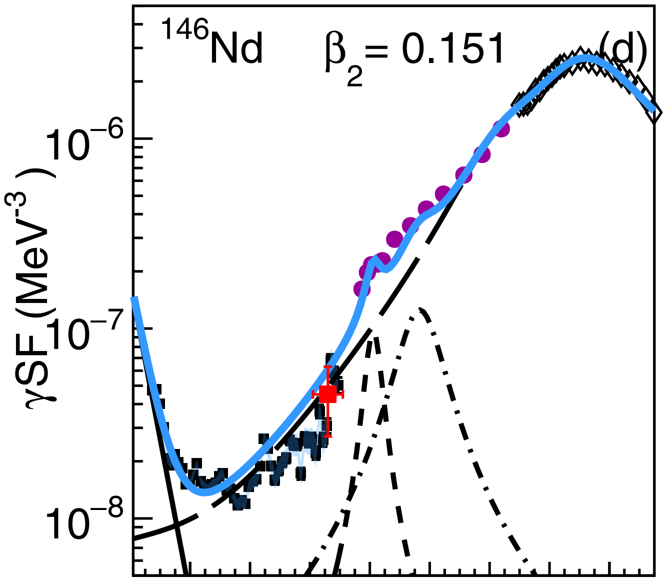
Yoram Alhassid (Yale University)

- Introduction: γ-ray strength functions and low-energy enhancement (LEE)
- Shell model Monte Carlo (SMMC) approach
- Analytic continuation: maximum entropy method (MEM)
- Prior strength function: static path approximation (SPA) plus RPA
- M1 strength functions in samarium and neodymium isotopes: the first theoretical identification of a LEE in heavy nuclei
- Conclusion and prospects.

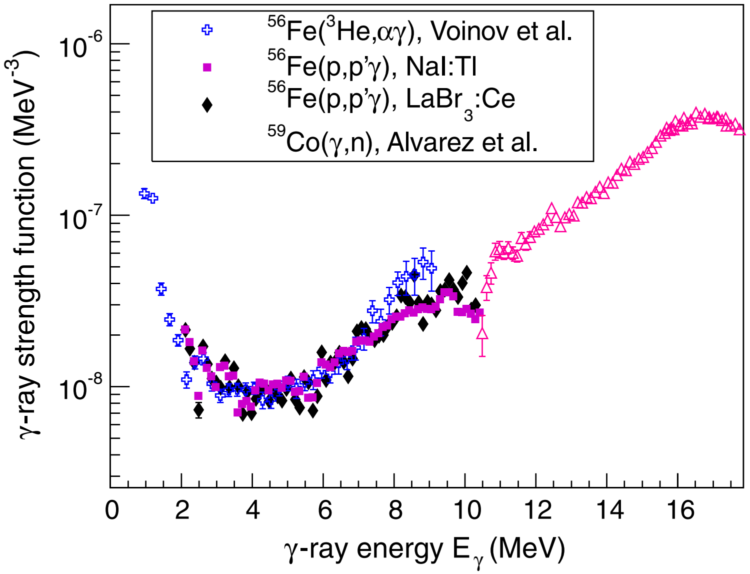
γ-ray strength functions

γ-ray strength functions (γSFs), along with level densities, are important input in the Hauser-Feshbach theory of compound nuclear reactions.

In recent years, a low-energy enhancement (LEE) or “upbend" was observed in the γSF of mid-mass nuclei and in a few rare-earth nuclei

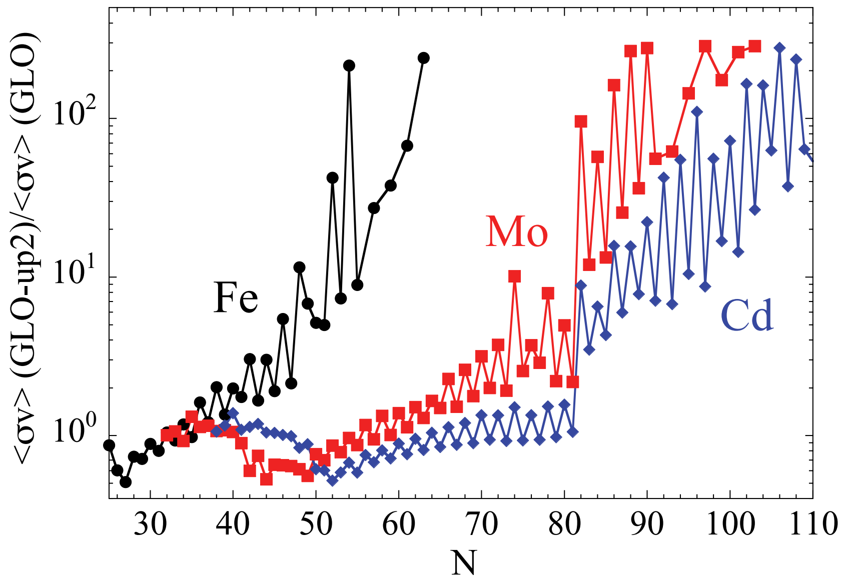


M. Guttormsen et al., PRC **106**, 034314 (2022)



A.C. Larsen et al., PRL **111**, 242504 (2013)

If the LEE persists in heavy neutron-rich nuclei, it can have significant effects on r-process nucleosynthesis by enhancing radiative neutron capture rates near the neutron drip line.



A.C. Larsen and S. Goriely, PRC 82, 014318 (2010)

The calculation of strength functions in the presence of correlations is a challenging many-body problem and microscopic approaches are limited:

- QRPA strength functions can often miss important correlations and require empirical modifications.
- The configuration-interaction (CI) shell model accounts for correlations but diagonalization methods are limited to spaces of dimensionality $\sim 10^{11}$.

CI shell model studies have attributed the LEE to the M1 γSF but they are limited to light and medium-mass nuclei.

The shell model Monte Carlo (SMMC method) enables microscopic calculations in spaces that are many orders of magnitude larger ($\sim 10^{30}$) than those that can be treated by conventional methods.

The shell model Monte Carlo (SMMC) method

Gibbs ensemble $e^{-\beta H}$ $(\beta = 1/T)$ can be written as a superposition of ensembles U_σ of *non-interacting* nucleons in time-dependent fields $\sigma(\tau)$

$$e^{-\beta H} = \int D[\sigma]\, G_\sigma U_\sigma$$

- The integrand reduces to matrix algebra in the single-particle space (of typical dimension ~ 100)

- The high-dimensional σ integration is evaluated by Monte Carlo methods.

SMMC is a powerful method to calculate physical observables (and in particular, level densities) in the presence of correlations but it cannot be used to calculate directly γSFs

The finite-temperature strength function of a transition operator O (e.g., E1,M1,...) is

$$S_O(\omega) = \sum_{i,f} \frac{e^{-\beta E_i}}{Z} |\langle f|O|i\rangle|^2 \, \delta(\omega - (E_f - E_i))$$

In SMMC, it is only possible to calculate imaginary-time response functions

$$R_O(\tau) = \langle O(\tau) O(0) \rangle$$

The response function $R_O(\tau)$ is the Laplace transform of the strength function

$$R_O(\tau) = \int_{-\infty}^{\infty} d\omega \, e^{-\tau\omega} S_O(\omega)$$

The inversion requires analytic continuation to real time and is numerically ill-defined (no unique solution)

We use the maximum entropy method (MEM): fitting to the SMMC response function while staying "sufficiently close" to a prior strength function

The success of the method depends on a good choice for a prior strength function
⇨ we use the static path plus random-phase approximation

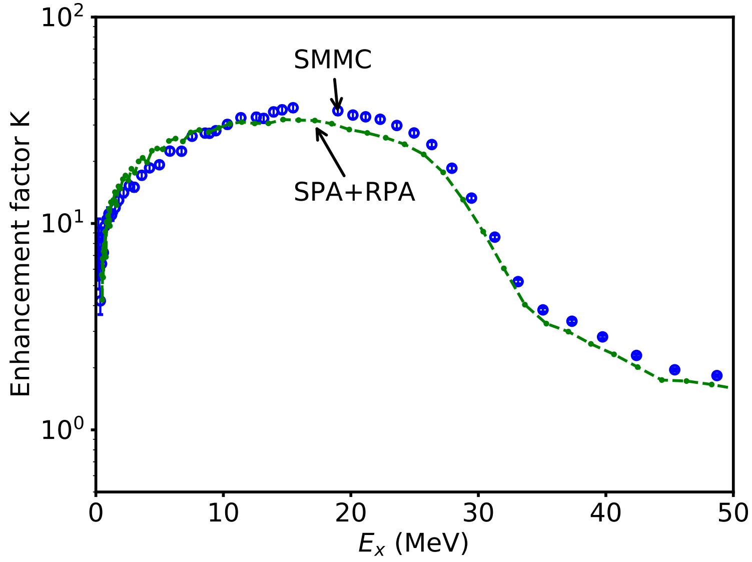
Static-path plus random-phase approximation (SPA+RPA)

Integrate over all static fluctuations of the mean field plus small time-dependent fluctuations (RPA) around each static fluctuation.

- Reproduces well the SMMC state densities

Enhancement factor of state density K (relative to mean field) in ^{162}Dy

P. Fanto and Y.A., PRC **103**, 064310 (2021)



The enhancement due to rotational collectivity is reproduced in the SPA+RPA

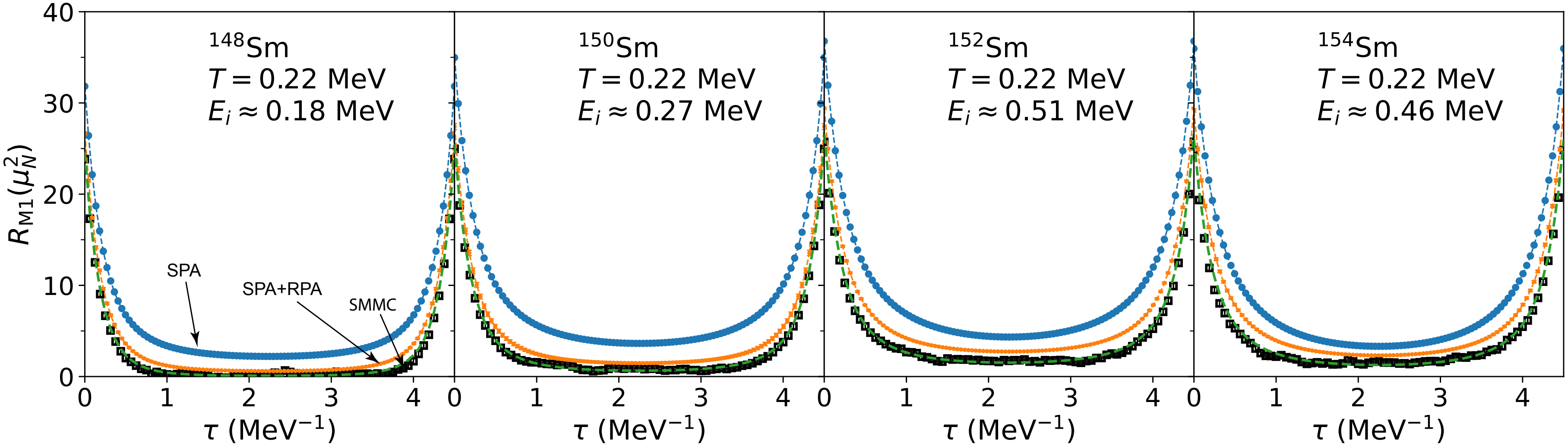
Application to heavy nuclei (lanthanides)

P. Fanto and Y.A., arXiv:2112.13772

Single-particle model space (using Woods-Saxon plus spin-orbit)

protons: 50-82 shell plus $1f_{7/2}$; neutrons: 82-126 shell plus $0h_{11/2}$ and $1g_{9/2}$.

M1 imaginary-time response functions in a chain of samarium isotopes

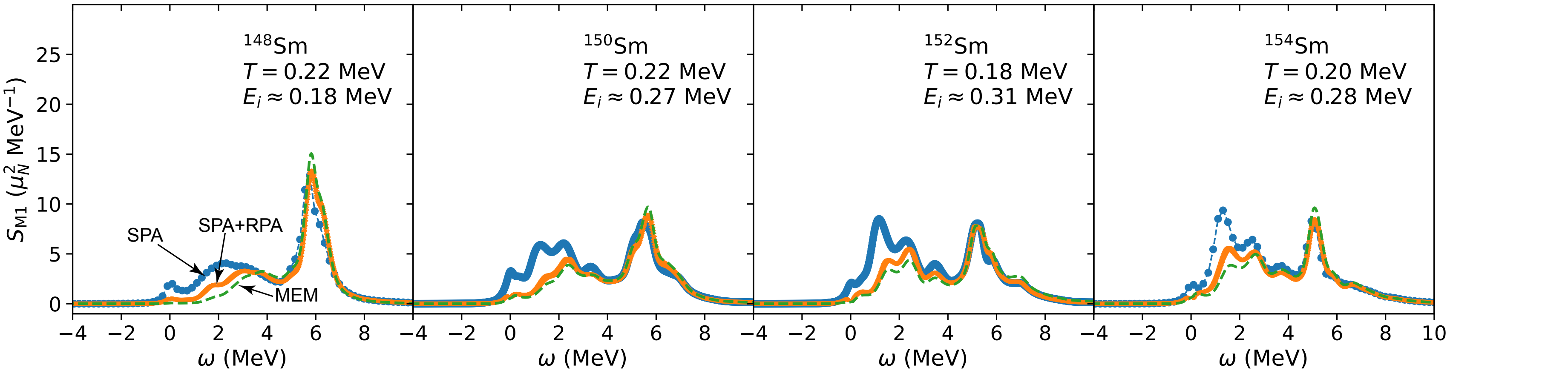


- The SPA+RPA response is close to the SMMC response

⇨ We use the SPA+RPA strength function as a prior and improve it using the MEM

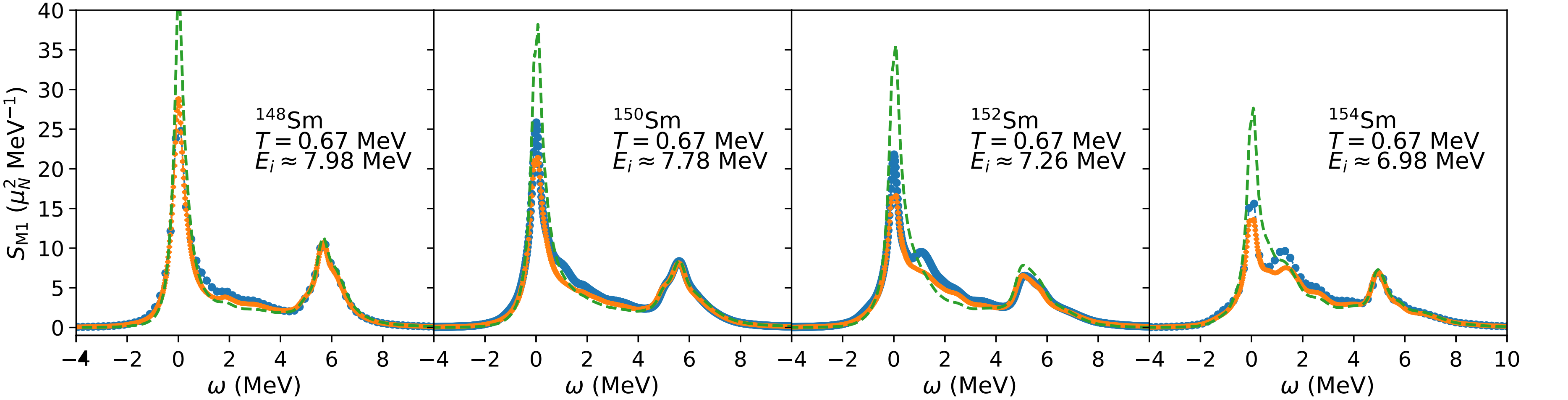
- The SPA is also a good starting point

Ground-state M1 strength functions



- Scissor mode around ~ 2 MeV and spin flip mode around ~ 6 MeV

M1 strength functions at an excitation energy of ~7-8 MeV (neutron resonance)



- The ω=0 peak is the LEE! – the first theoretical identification in heavy nuclei
- Strength from the LEE is transferred to the scissors mode (built on top of excited states) in the crossover from spherical to deformed nuclei

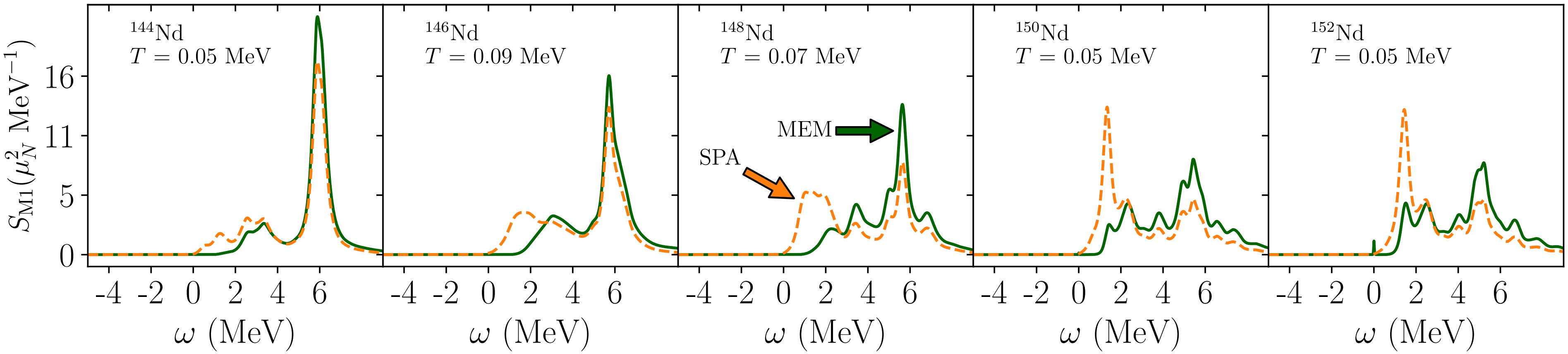
Application to a chain of neodymium isotopes

A. Mercenne, P. Fanto and Y.A. (2023)

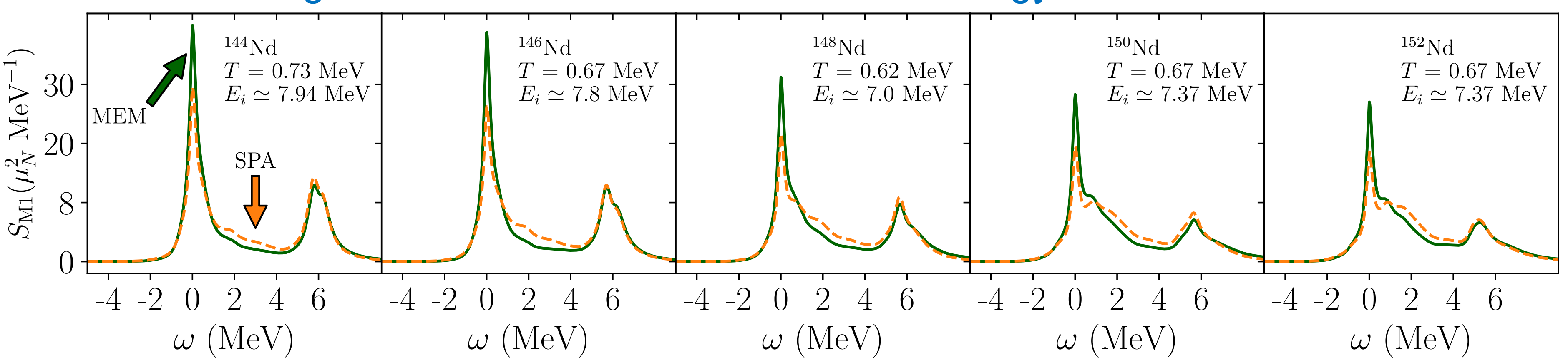
For the full SMMC interaction, the SPA+RPA is time-consuming

⇨ use the SPA strength function as a prior

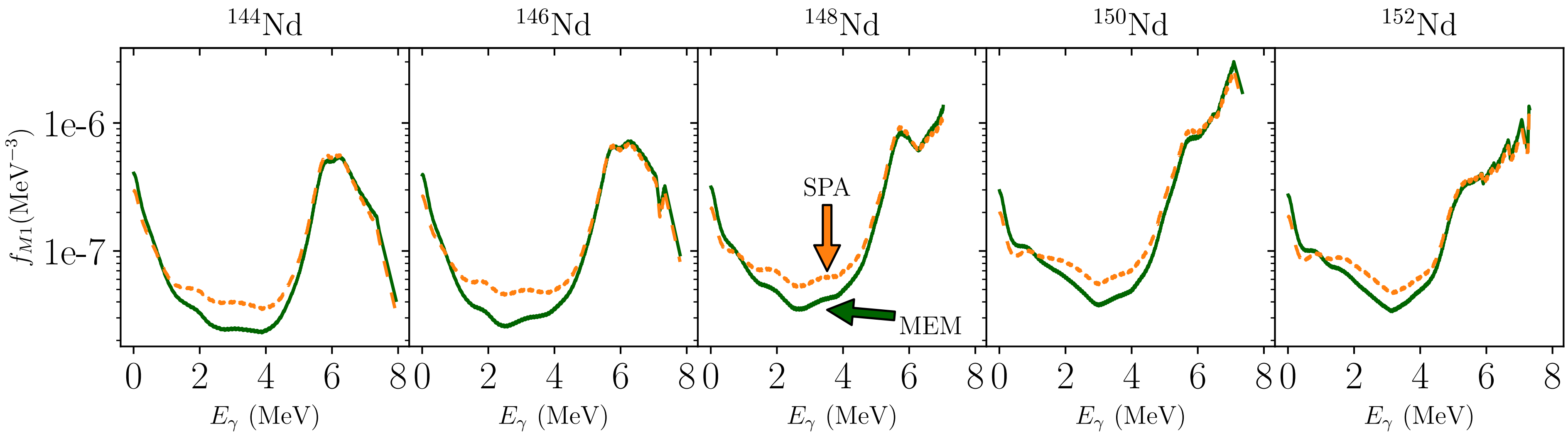
Ground-state M1 strength functions



M1 strength functions at neutron resonance energy



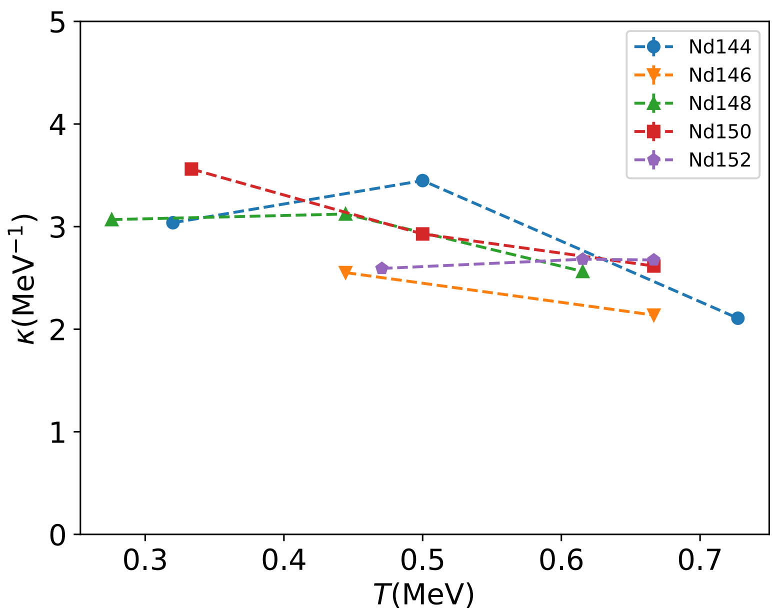
Converting the strength function S_{M1} to the strength function f_{M1} for decay



$$f_{M1}(E_\gamma) = a\frac{\rho(E_i)}{\rho(E_i - E_\gamma)} S_{M1}(\omega = -E_\gamma)$$

The LEE is empirically parametrized by

$$f_{M1}(E_\gamma) \propto e^{-\kappa E_\gamma}$$



- Slope κ depends weakly on temperature

Conclusion

- We introduced a reliable method to calculate strength functions for heavy nuclei in the presence of correlations by combining two many-body methods: SMMC and the SPA+RPA (or SPA)
- We made the first theoretical identification of a low-energy enhancement (LEE) in heavy nuclei within the framework of the CI shell model
- We observed the emergence of a structure consistent with a scissors mode built on top of excited states as neutron number increases

Prospects

- Calculate γSF in odd-mass isotopes
- The experiments cannot separate the E1 and M1 components of the γSF - A detailed comparison with experiments requires the calculation of E1, and thus the inclusion of another major shell
- Extend the approach to heavier nuclei (e.g., actinides) and to neutron-rich nuclei