Cold atomic Fermi gases in two dimensions: superfluidity and pseudogap effects in the strongly interacting regime

Yoram Alhassid (Yale University)

Shasta Ramachandran
(Yale U.)

Scott Jensen (U. Illinois,
Urbana-Champaign)

2D Cold atomic Fermi gases in the strongly interacting regime

- Introduction: Interacting Fermi gas in three (3D) and two (2D) dimensions
- Canonical-ensemble auxiliary-field Monte Carlo (AFMC) method
- Lattice formulation and continuum limit
- Pseudogap effects
- The contact
- Conclusion and outlook

Introduction

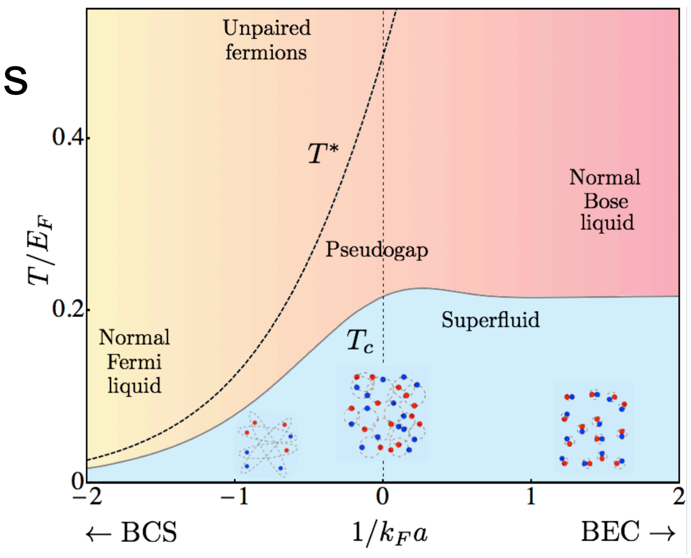
Consider a two-component (spin up/down) fermionic atoms interacting with a short-range interaction $V_0\delta(\mathbf{r}-\mathbf{r}')$ characterized by a scattering length a .

Three spatial dimensions (3D)

- A two-particle bound state for $a > 0$
- The scattering amplitude at low momentum k is

$$f(k) = \frac{1}{-1/a - ik}$$

- A crossover from BCS for $(k_F a)^{-1} \sim -\infty$ to BEC for $(k_F a)^{-1} \sim +\infty$
- Phase transition to a superfluid below a critical temperature T_c



Randeria and Taylor, 2014

Of particular interest is the limit of strongest interaction $a \to \infty$ or $(k_F a)^{-1} = 0$
-- The unitary Fermi gas

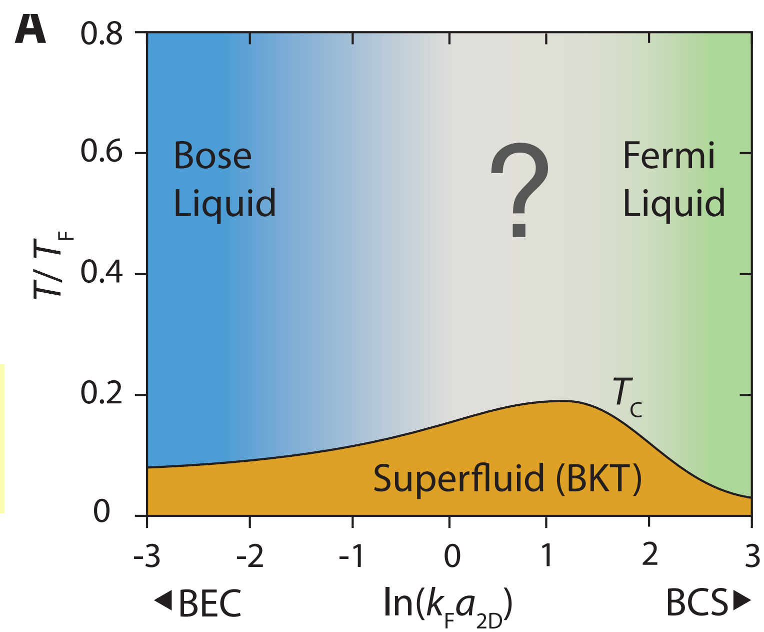
Of interest for high-T_c superconductivity, nuclear matter, and other strongly interacting Fermi systems

Two spatial dimensions (2D)

- There is a bound two-particle state for arbitrarily weak interaction strength
- The scattering amplitude at low momentum k is given by

$$f(k) = \frac{2\pi}{[\ln(2/kae^{\gamma}) + i\pi/2]}$$

- A crossover from BEC to BCS as a function of $\ln(k_F a)$



Murthy et al, Science 2018

- The phase transition to superfluidity is of the Berezinskii-Kosterlitz-Thouless type

- Strong coupling regime: $\ln(k_F a) \sim 1$

- A challenging non-perturbative many-body problem

Many theoretical methods have been used to study the thermodynamics of the strongly coupled Fermi gas:

Strong-coupling theories:

Early theories: Leggett (1980), Nozieres and Schmidt-Rink (1985)

T-matrix approaches

Self-consistent Luttinger-Ward theory

...

Quantum Monte Carlo methods:

Lattice diagrammatic Monte Carlo (LDMC)

Bold diagrammatic Monte Carlo (BDMC)

Auxiliary-field quantum Monte Carlo (AFMC)

...

Canonical ensemble auxiliary-field Monte Carlo (AFMC) method

AFMC is based on the Hubbard-Stratonovich transformation, which describes the Gibbs ensemble $e^{-\beta H}$ at inverse temperature $\beta = 1/T$ as a path integral over time-dependent auxiliary fields $\sigma(\tau)$

$$e^{-\beta H} = \int D[\sigma]\, G_\sigma U_\sigma$$

G_σ is a Gaussian weight and U_σ is a propagator of *non-interacting* particles moving in external auxiliary fields $\sigma(\tau)$

- The integrand reduces to matrix algebra in the single-particle space.

The high-dimensional integration over σ is evaluated by importance sampling.

We implemented the canonical ensemble by exact particle-number projection.

Recent review of AFMC: Y. Alhassid, in *Emergent Phenomena in Atomic Nuclei from Large-Scale Modeling*, ed. K.D. Launey (World Scientific 2017)

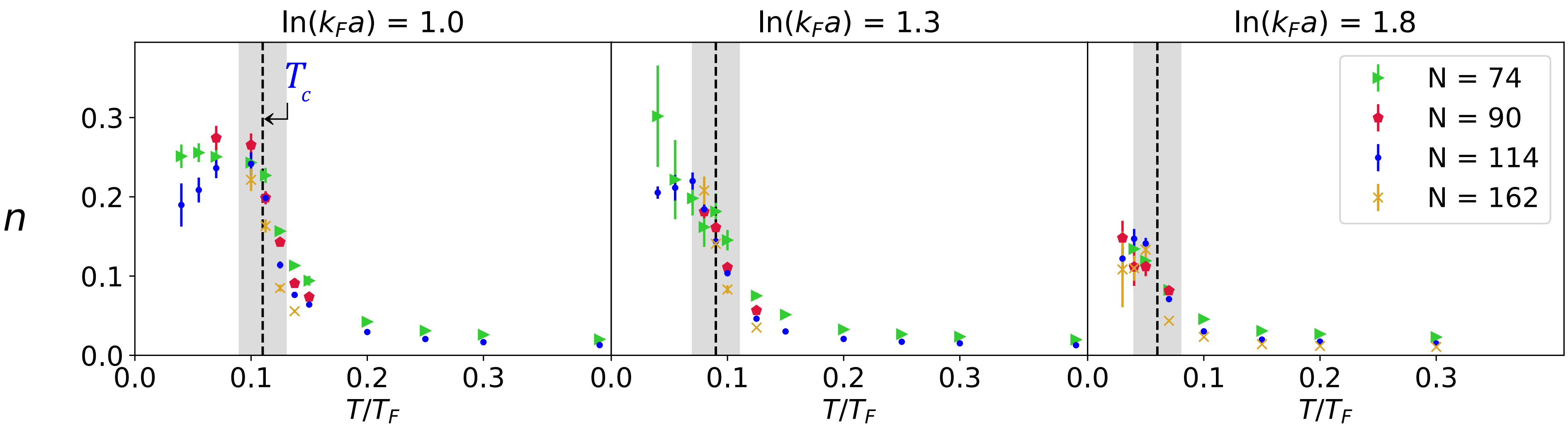
Superfluidity in 2D

We studied 3 couplings in the strongly interacting regime: $\ln(k_F a) = 1,\ 1.3,\ 1.8$

There is a phase transition to superfluidity below a critical temperature T_c

Condensate fraction n :

Calculated from $n = \lambda_{\max} / (N/2)$, where $\lambda_{\max}$ is the largest eigenvalue of the pair correlation matrix $\langle a^\dagger_{\mathbf{k}_1\sigma_1} a^\dagger_{\mathbf{k}_2\sigma_2} a_{\mathbf{k}_4\sigma_4} a_{\mathbf{k}_3\sigma_3} \rangle$



We determine the critical temperature T_c by using finite-size scaling of n that is suitable to a BKT phase transition

Pseudogap effects

Are there signatures of a pseudogap regime above the critical temperature in which pairing correlations survive?

Spin susceptibility

$$\chi \propto \beta \langle (N_\uparrow - N_\downarrow)^2 \rangle$$

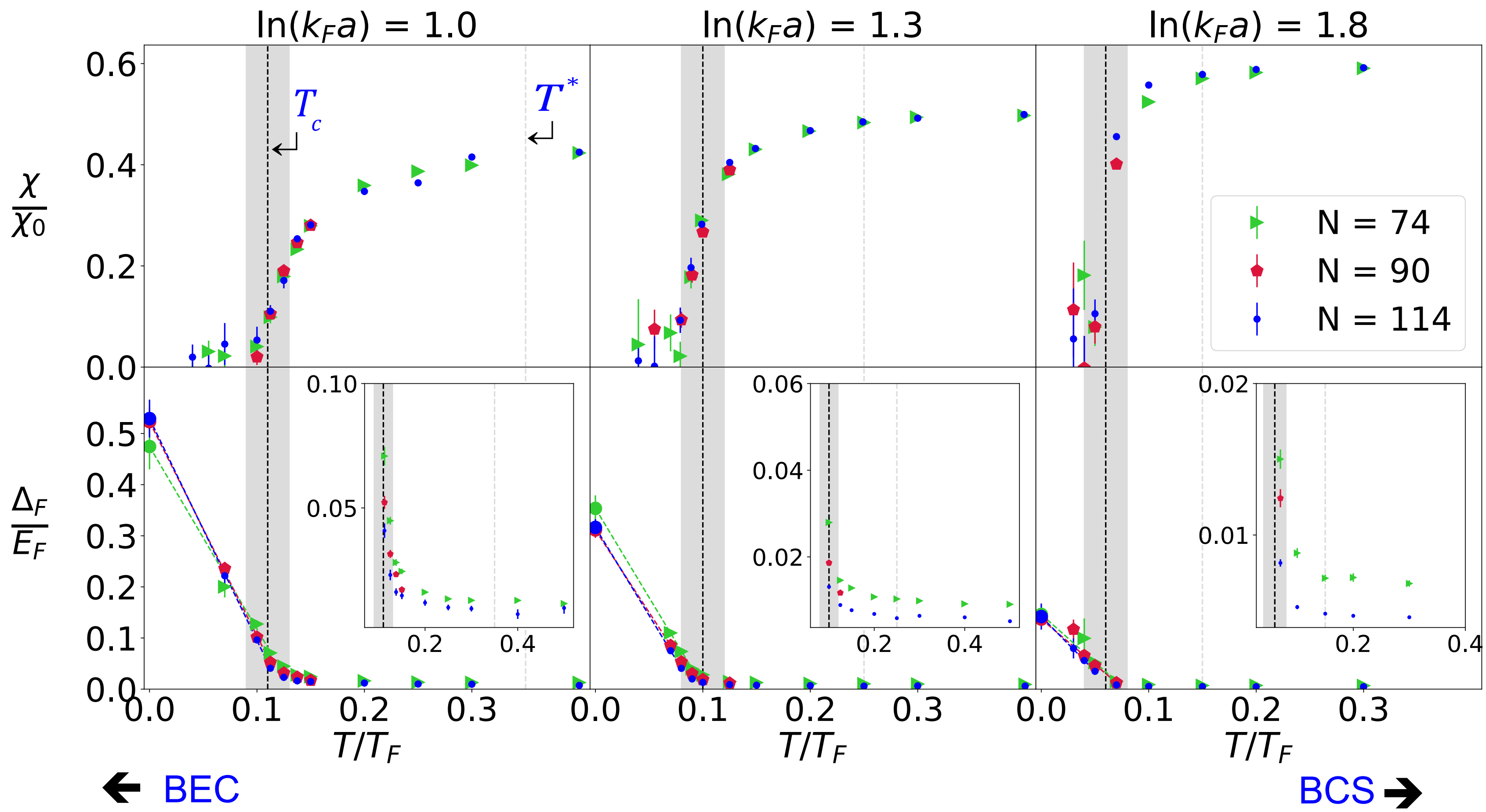
Pairing correlations suppress the spin susceptibility

Free energy gap

$$\Delta_F = [2F(N_\uparrow, N_\downarrow - 1) - F(N_\uparrow, N_\downarrow) - F(N_\uparrow - 1, N_\downarrow - 1)]/2$$

-- requires the canonical ensemble

Spin susceptibility (top) and free energy gap (bottom) vs. temperature



The regime $T_c < T < T^*$ in which the spin susceptibility is suppressed (spin gap) is substantial for $\ln(k_F a) = 1$ and becomes narrower on the BCS side

The free energy gap increases as T decreases towards T_c in the spin gap regime

The contact C

A fundamental thermodynamic property of quantum many-body systems with short-range interactions

- The contact C describes the short-range pair correlation at distance $r \to 0$

3D: $\langle n_\uparrow(r) n_\downarrow(0) \rangle \sim \frac{C}{4\pi r^2}$ 2D: $\langle n_\uparrow(r) n_\downarrow(0) \rangle \sim \frac{C}{(2\pi)^2} \ln^2 r$

- Characterizes the tail of the momentum distribution $n_\sigma(k) \sim \frac{C}{k^4}$

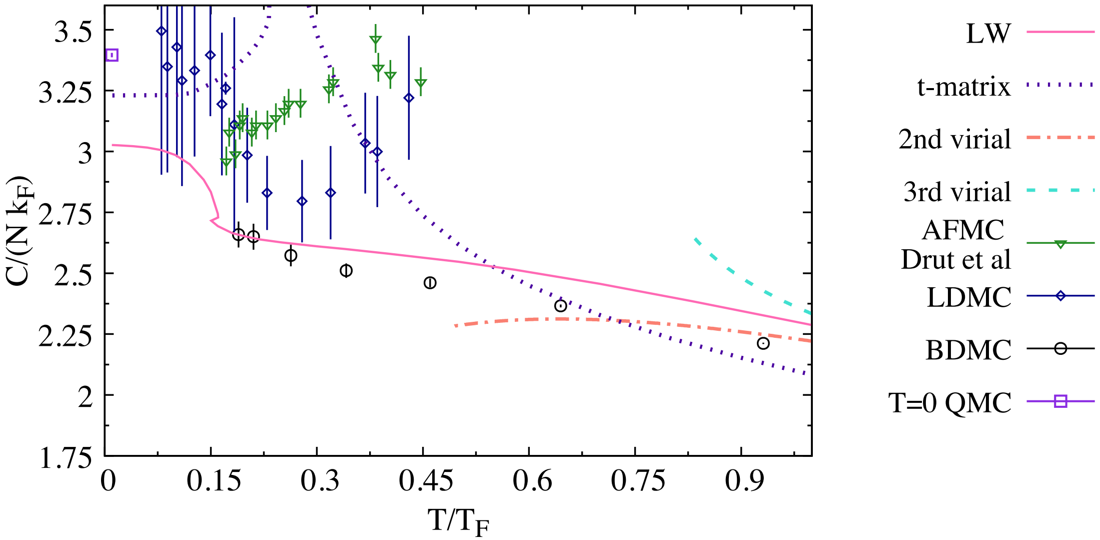
- Characterizes the high-frequency tail of the shear viscosity spectral function

- Can be expressed as the adiabatic derivative of the energy with respect to the inverse scattering length (3D) or $\ln a$ (2D)

3D: $C = \frac{4\pi m}{\hbar^2} \frac{\partial E}{\partial(-1/a)}$ 2D: $C = \frac{4\pi m}{\hbar^2} \frac{\partial E}{\partial \ln a}$

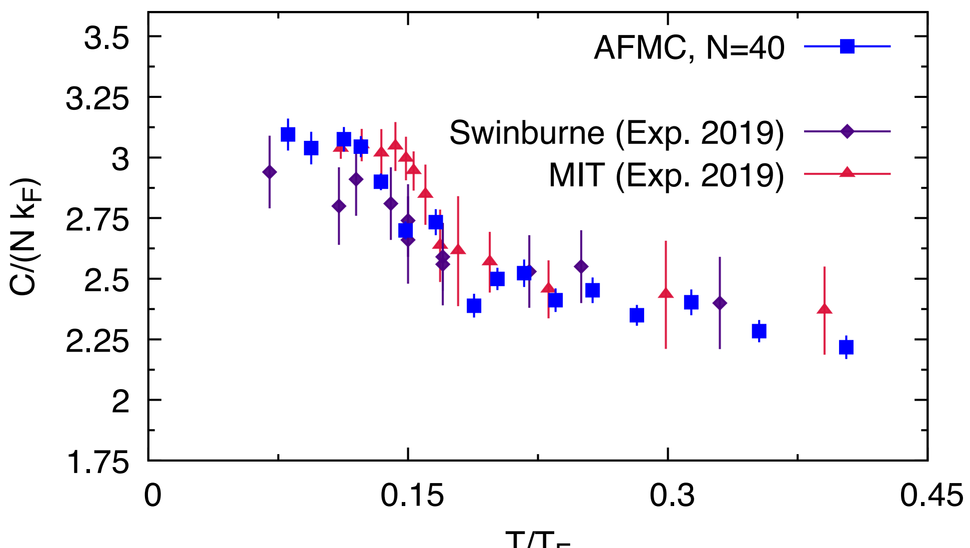
The measurement and theoretical calculation of the temperature dependence of the contact has been a major challenge in the last decade

Contact in the unitary Fermi gas (3D)



Theoretical calculations differ widely, even on a qualitative level.

--Many of the strong coupling theories are based on uncontrolled approximations



Our continuum limit AFMC results provide the best quantitative agreement with the recent precision experiments.

S. Jensen, C.N. Gilbreth, and Y. Alhassid, Phys. Rev. Lett. **125** (2020)

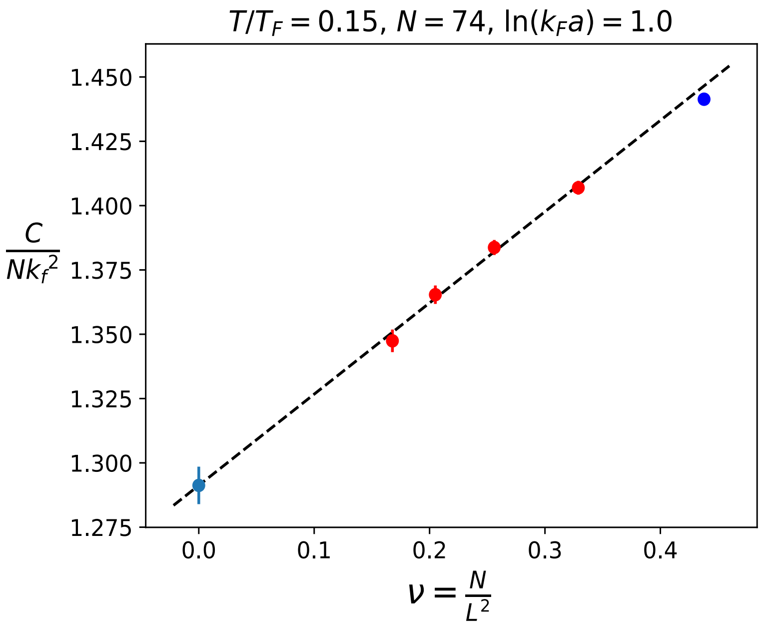
Continuum limit

S. Jensen, C.N. Gilbreth, and Y. Alhassid, Phys. Rev. Lett. **125** (2020)

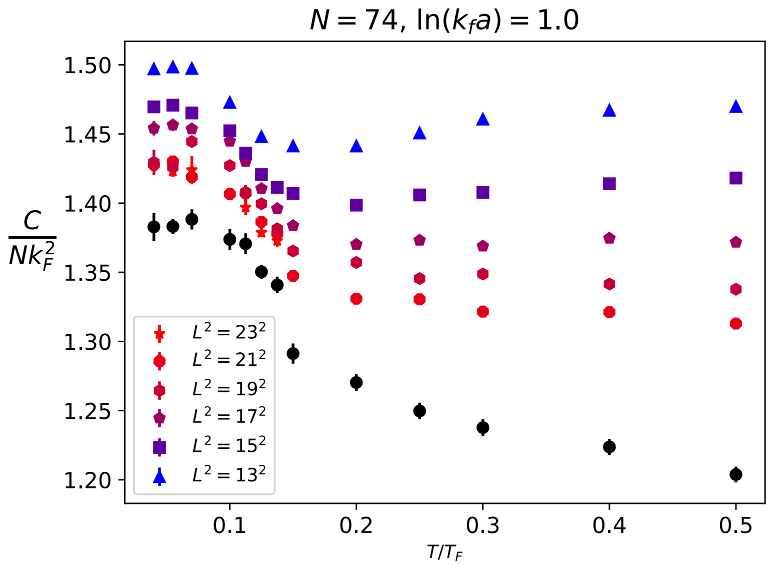
The continuum limit (filling factor $\nu \to 0$) is major challenge requiring AFMC calculation on large lattices.

We introduced a novel method that ignores the (almost) unoccupied single-particle states, enabling large lattice calculations (Comp. Phys. Comm. 2021)

Example: contact in 2D



In 2D, the extrapolation is linear in ν



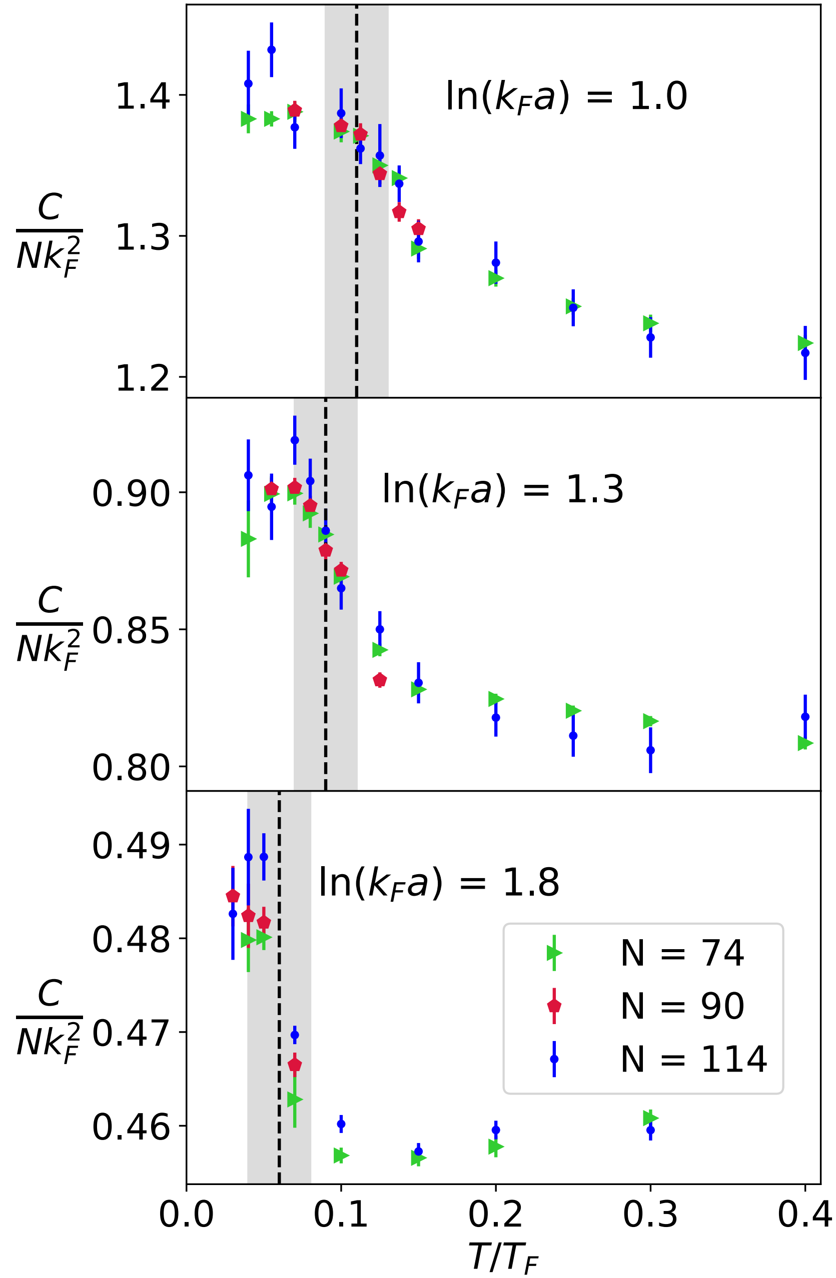
The contact is sensitive to the filling factor ν, and the extrapolation is crucial

Contact in 2D

Calculated from $C \propto \langle V \rangle$

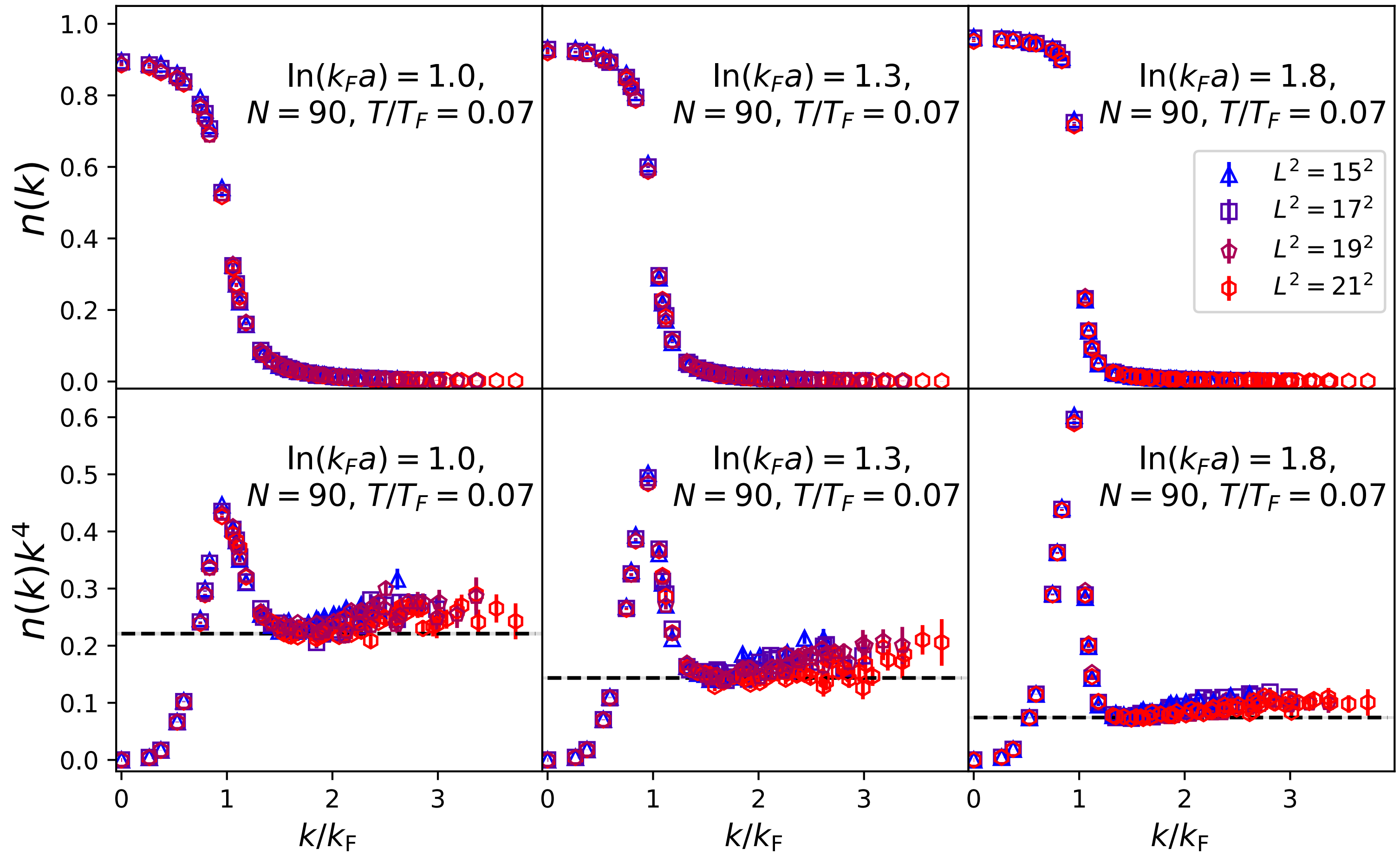
A rapid increase below T_c for all couplings

Magnitude becomes smaller towards the BCS side



Single-particle momentum distribution

For large k: $n(k) = \frac{C}{k^4}$



Conclusion

- Precision thermodynamics of the interacting Fermi gas has been a major challenge to both experimentalists and theorists.
- Most theoretical methods use uncontrolled approximations and lead to widely different results
- We performed accurate auxiliary-field Monte Carlo (AFMC) calculations on the lattice, eliminating systematic errors associated with finite lattice spacing
- We find a significant pseudogap regime for the 2D Fermi gas at $\ln(k_F a) = 1$ that becomes narrower towards the BCS side
- The contact exhibits rapid increase below T_c

Outlook

- Calculate dynamical observables in AFMC: spectral weight, shear viscosity,...
- Carry out precision experiments in a uniform trap (3D and 2D).